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FLOW PAST FORWARD-FACING SMALL STEP

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Uniform subsonic or supersonic laminar flow of a viscous fluid past a flat plate is considered. A small two-dimensional roughness element is present on the flat plate surface at a distance l from the leading edge. The solution to Navier—Stokes equations is developed for the case when the characteristic Reynolds number $Re_0 = \rho_0 u_0 l/\mu_0 = \varepsilon^{-2}$ tends to infinity (ρ_0, u_0, μ_0 are the density, velocity, and the coefficient of dynamic viscosity in the undisturbed free stream). In what follows, only nondimensional quantities will be used and for this purpose the reference quantities are: l for length, u_0 for velocity, ρ_0 for density, $\rho_0 u_0^2$ for pressure, u_0^2 for enthalpy, $\rho_0 u_0 l$ for the stream function, and μ_0 for the coefficient of dynamic viscosity. Systematic studies on the flow past small roughness over the surface of a body with characteristic transverse and longitudinal dimensions a and b ($\varepsilon^2 \ll a \lesssim \varepsilon$, $a \lesssim b \ll 1$) have been done in [1, 2], where, in particular, it has been shown that the flow near a roughness with $a \sim b \sim O(\varepsilon^{3/2})$ in the first approximation as $\varepsilon \to 0$, is described by Navier—Stokes equations for incompressible fluid, the velocity profiles and enthalpy in the external flow are sheared and the critical similarity parameter is the local Reynolds number $Re = \rho_w A a_1^2/\mu_w$ (the index w refers to the values at the flat plate surface in undisturbed boundary layer), A is the shear stress at the flat plate surface in undisturbed boundary layer, $a = \varepsilon^{3/2} a_1$, $a_1 \sim O(1)$. For Re, it is possible to obtain the following estimate [3]: $Re \sim Re_0^{1/2}(a/\varepsilon)^2$, from which it follows that as $a/\varepsilon \ll 1$ and $Re_0 \lesssim 10^6$ (i.e., for real and practically significant values of Re_0) the value of Re cannot exceed a few tens. Solutions to Navier—Stokes equations for the incompressible shear flow past a roughness on a body surface with $Re \lesssim 100$ are obtained in [4-6]. One of the distinctive features of these solutions is their existence at Re = 0 [5, 6], i.e., solutions of Navier—Stokes equations have been obtained for plane flows. Besides, even at Re = 0 separated zones have been observed in the flow field. The damping of disturbances far behind such roughness is also very typical and its study can be made with an analysis of the boundary layer equations along with the local condition for the interaction with the subsonic wall layer of the undisturbed boundary layer [7, 8].

It is useful to mention that the flow past roughness with $\varepsilon^{3/2} \ll a \sim b \ll \varepsilon$ in the first approximation as $\varepsilon \to 0$ is described by Euler equations for incompressible fluid with an external shear flow [1, 2].

Let there be a rectangular step on a flat plate with a characteristic height $a \sim O(\varepsilon^{3/2})$ and a characteristic length $\varepsilon^{3/2} \ll b \ll \varepsilon^{3/4}$. As shown in [1, 2], the flow past such a roughness is described by linearized incompressible boundary-layer equation with linearized local conditions for the interaction with the subsonic wall layer of the undisturbed boundary layer. It has been obtained in [8] that on the surface of such a step as one moves away from its face the disturbances in heat flux Δq and shear stress $\Delta\tau$ damp out at the following rate with respect to their values in the undisturbed boundary layer at the plate surface

$$\Delta q \sim \Delta\tau \sim x^{-1/3} \ (x \to \infty) \tag{2.1}$$

(i.e., damping of disturbances q and τ is very weak), and pressure disturbances $p < 0$ increase

$$|p| \sim x^{1/3} \ (x \to \infty). \tag{2.2}$$

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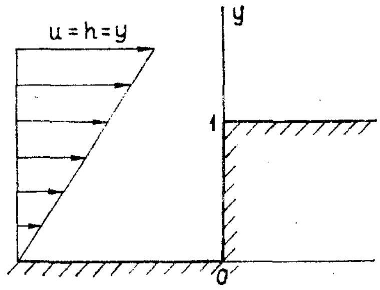

Fig. 1

In order to study the flow in the neighborhood of the front face of the step (forward facing rectangular step), it is necessary to consider the region with characteristic dimension $x \sim y \sim O(\varepsilon^{3/2})$, for which the Navier–Stokes equations are valid, and the external flow is the near-wall subsonic flow of the undisturbed boundary layer (Fig. 1):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2.3}$$
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),\quad u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=\frac{1}{\mathrm{Re}\,\sigma}\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right),$$
$$u_w=v_w=h_w=0,\quad u\to y,\quad h\to y\quad (x^2+y^2\to\infty).$$

Here all the linear dimensions are referred to the height of the step a_1, velocity and enthalpy disturbance with respect to their value at the surface of the body, i.e., to their values at a height a_1 in the wall layer of the undisturbed boundary layer Aa_1 and Ba_1 (B is the heat flux at the plate surface in the undisturbed boundary layer), pressure and stream function are referred to $\rho_w A^2 a_1^2$ and $\rho_w A a_1^2$; σ is the Prandtl number. With these quantities, the shear stress τ and heat flux q in the undisturbed boundary layer at the flat plate surface equal one.

The solution of the boundary-value problem (2.3) at Re $\leqslant$ 100 makes it possible to study aerodynamic heating of the practically important element, viz., the small rectangular step on the surface of the body, makes it possible to study the nature of the separated flow near such a typical configuration.

It is convenient to introduce new dependent variables for the numerical solution of the boundary-value problem (2.3), viz., for the fluctuation of stream function, enthalpy, and vorticity with respect to their values in the mean shear flow

$$\psi = y^2/2 + \varphi,\ h = y + g,\ \partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 = 1 - \zeta,$$

in which Navier–Stokes equations (2.3) take the form

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=-\zeta,\ \frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}=\mathrm{Re}\left[\frac{\partial}{\partial x}\left(\zeta\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(\zeta\frac{\partial\psi}{\partial x}\right)\right], \tag{2.4}$$
$$\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=\mathrm{Re}\,\sigma\left[\frac{\partial}{\partial x}\left(g\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(g\frac{\partial\psi}{\partial x}\right)-\frac{\partial\psi}{\partial x}\right].$$

Instead of the pressure p the function $D = \mathrm{Re}(p + (u^2 + v^2)/2 - y^2/2 - \varphi)$ is introduced and its behavior in the flow field is described by the equation

$$\frac{\partial^2 D}{\partial x^2}+\frac{\partial^2 D}{\partial y^2}=-\mathrm{Re}\left[\frac{\partial}{\partial x}\left(\zeta\frac{\partial\psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\zeta\frac{\partial\psi}{\partial y}\right)\right]. \tag{2.5}$$

Obviously, on the surface of the body $D \equiv p$.

Boundary conditions on the surface of the body for the function φ(x, y) and g(x, y) have the form

$$\varphi(x,0)=0,\ \varphi(0,y)=-y^2/2,\ \varphi(x,1)=-1/2, \tag{2.6}$$
$$g(x,0)=0,\ g(0,y)=-y,\ g(x,1)=-1.$$

For the function ζ(x, y) the usual approximate conditions of first-order accuracy [9] are used

(2.7)

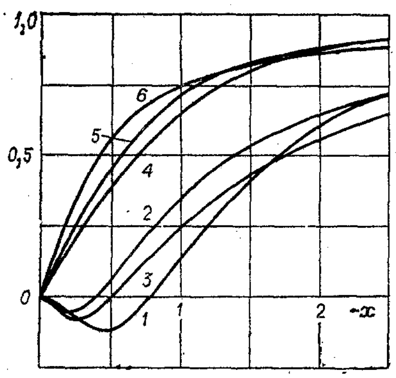

Fig. 2

$$\zeta(x,\ 0) = -2\varphi(x,\ \Delta y)/\Delta y^2 + O(\Delta y),$$
$$\zeta(0,\ y) = 1 - 2[\varphi(-\Delta x,\ y) + y^2/2]/\Delta x^2 + O(\Delta x),$$
$$\zeta(x,\ 1) = -2[\varphi(x,\ 1 + \Delta y) + 1/2 + \Delta y]/\Delta y^2 + O(\Delta y);$$

for the function D(x, y) only normal derivatives are specified,

$$(\partial D/\partial y)(x,\ 0) = (\partial \zeta/\partial x)(x,\ 0),\ (\partial D/\partial x)(0,\ y) = (\partial \zeta/\partial y)(0,\ y),$$
$$(\partial D/\partial y)(x,\ 1) = (\partial \zeta/\partial x)(x,\ 1).$$

In deriving the outer boundary conditions it is useful to utilize the condition that, as already mentioned above, when $x^2 + y^2 \to \infty$, the given flow becomes inviscid. It is easy to observe that the function

$$\psi(x,\ y) = y^2/2 - \arg(y/x)/2\pi \tag{2.8}$$

describes a shear flow of an ideal fluid past a step. Hence, the expression (2.8) can be used as the outer boundary condition for the problem (2.3). The outer boundary conditions for the function $\varphi(x, y)$, $\zeta(x, y)$ and $g(x, y)$ were used in the form

$$\varphi \sim \arg(y/x),\ \zeta \sim xy/(x^2 + y^2)^2,\ g \sim \arg(y/x)/y\ (x^2 + y^2 \to \infty). \tag{2.9}$$

To compute boundary conditions at $y \to \infty$ for the function D(x, y), the following equation was integrated

$$\partial D/\partial x = -\mathrm{Re}\,\zeta\partial\varphi/\partial x - \partial\zeta/\partial y \tag{2.10}$$

with initial condition

$$D(-\infty,\ \infty) = 0, \tag{2.11}$$

as $x \to \pm\infty$ the normal derivatives $\partial D/\partial_x$ from Eq. (2.10) were specified.

A nonuniform rectangular difference grid of dimension 43 × 32 was specified in the flow field and its concentration increased in geometric progression with index k = 5/6 as the surface of the body was approached. Computations were made at minimum step sizes for the difference grid $\Delta x = \Delta y = 0.067$, the outer boundary of the computational field was located at $x \approx \pm 15$ and $y \approx 16$ (at the face of the step for such a breakup of the computational field there were 10 intervals of difference grid).

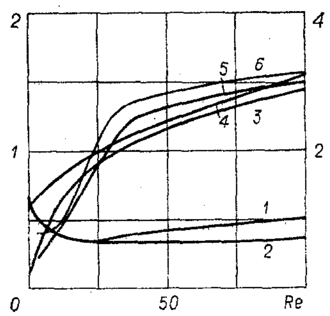

Fig. 3

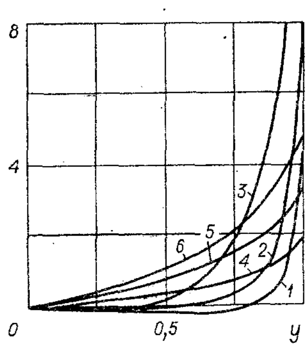

Fig. 4

The boundary-value problems (2.4), (2.6), (2.7), and (2.9) were approximated by the first-order accurate difference [10] which was solved by an iterative procedure. The method of variable direction [11] was used at each iteration. Iterations were completed when the difference in the values of shear stress or heat flux at the surface of the body between two consecutive iterations was less than 0.1%.

Computational results have been obtained in the range of local Reynolds number Re = 0-100. Prandtl number was assumed equal to 0.71.

Computational results show that the fluid flow ahead of the step is decelerated, shear stress τ and heat flux q at the surface of the body are decreased. As $x \gtrsim -1$ there is a flow separation, there is a separated zone, and the reattachment of the flow takes place close to the center of the step face.

The values of τ and q on the face of the step sharply increase toward the upper edge of the face of the step. At the upper side of the step τ and q initially fall very considerably with distance from the face of the step but then their change is very small and the disturbances are propagated far downstream. The very neighborhood of the upper edge of the face of the step does not form separated zones. Such a flow field is maintained in the entire range of Reynolds number under study.

Figure 2 shows the distribution of τ for Re = 0; 31.6; 100 (curves 1-3) and q for Re = 0; 10; 100 (curves 4-6) ahead of the step. It is seen that disturbances from the step are propagated quite far upstream, the propagation of disturbances is intensified with increase in Re, the extent of the separated zone varies nonmonotonically as a function of Re. The separated zone is the largest at Re = 0, its extent x_1 = 0.77 and height y_1 = 0.775. Initially with increase in Re the dimensions of the separated zone decrease rapidly with $x_1 \approx y_1$. At Re $\approx$ 20 the separation zone has minimum dimensions $x_1 \approx y_1 \approx 0.35$. With further increase in Re x_1 begins to rise and the value of y_1 practically remains constant (see curves 1, 2 in Fig. 3, left axis of ordinates). An increase in Re practically corresponds to an increase in the external flow velocity (more correctly, an increase in its gradient A). Hence at small values of Re, its incease leads to delay of separation since pressure disturbances $\sim Re^2$ are still small. With further increase in Re pressure disturbances rise very strongly and the separated zone begins to increase.

The distributions of τ at Re = 0; 3.16; 31.6 (curves 1-3) and q at Re = 0; 31.6; 100 (curves 4-6) along the face of the step are shown in Fig. 4. The neighborhood of the upper edge of the face of the step is a singular region for the given flow and the values of τ and

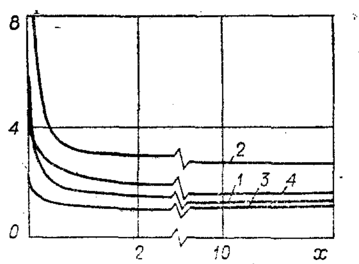

Fig. 5

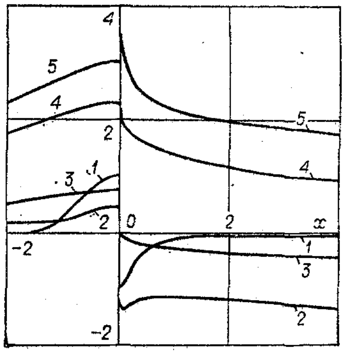

Fig. 6

q there do not have any practical meaning. However, the integral values $\int_0^1 \tau dy$ and $\int_0^1 q dy$ may have great practical interest, being the total Stein friction drag and heat flux acting on the face of the step (curves 3, 4 in Fig. 3, the right and the left axis of ordinates).

The damping characteristics of the disturbance downstream are shown in Fig. 5, where the distribution of τ at Re = 0; 100 (curves 1, 2) and q at Re = 0, 100 (curves 3, 4) behind the step are shown. It is seen that close to the face of the step at $x \leqslant 1.5$ skin friction and heat flux vary strongly but farther away from the face of the step their variation is small. Such a behavior of τ and q at $x \gg 1$ is found to be in quantitative agreement with the damping characteristic of disturbances (2.1) described earlier. We note that at Re = 100, for example, q exceeds its value in undisturbed boundary layer at the plate surface by nearly 1.5 times even at $x \geqslant 10$.

A fivefold increase in the computational accuracy, the use of second-order accurate conditions for vorticity at the surface of the body (instead of conditions (2.7)), or the use of a smaller difference grid (20 intervals of difference grid on the face of the step, minimum step size of the difference grid $\Delta x = \Delta y = 0.019$) do not increase the accuracy of the computed results beyond the accuracy of the approximation used in the computational scheme. It is true that computation with smaller grids at the lower corner of the step showed one more circulation zone having a size almost twice the interval of the computational grid in terms of the length and height.

The characteristics of the computation of pressure using the known numerical solution for the stream function and vorticity are described in detail in [12]. In the present work, it is proposed to use the values of the function D(x, y) obtained by integrating Eq. (2.10) with initial condition (2.11) as the boundary conditions at $y \to \infty$ which makes it possible to correctly obtain the solution of Eq. (2.5). Obviously, in integrating (2.10) errors of approximation occur but from an analysis of the computed results for $\zeta(x, y)$ it follows that an $Re \geqslant 1$ and $y \geqslant 6$ $\zeta(x, y) \equiv 0$, and then the integration of (2.10) with the initial condition (2.11) gives $D(x, y) = 0$ ($Re \geqslant 1$, $y \to \infty$).

The differential equation (2.5) in the first difference grid was approximated by difference equations using central differences; the method used was the same as that used to compute $\varphi(x, y)$ or $\zeta(x, y)$. The pressure distribution at Re = 0: 3.16; 10; 31.6; 100 (curves 1-5) along the surface of the step are shown in Fig. 6 (at Re = 0 the quantity Re p is finite); it is clearly seen that there is a significant increase in the upstream propagation of disturbances with increase in Re. The pressure continues to rise with an upward movement along the face of the step and while crossing over the upper side of the step there is a sudden drop in pressure and then for $x \gg 1$ it continues to fall gradually. Unfortunately, computed results do not confirm the asymptotic nature of the variation in pressure (2.2) (for this it is necessary to considerably increase the dimensions of the computational field downstream). The relations between pressure at the separation and reattachment points and Re are shown in Fig. 3 (curves 5, 6, right axis of ordinates).

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UNSYMMETRIC CORNER FLOWS

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Streamwise flow past corners is characterized by the development of secondary flows in the neighborhood of the bisecting plane [1-4]. These streamwise vortices in the boundary layer were determined by Prandtl [5] as secondary flows of the second type, i.e., flows where the velocity gradients $\partial/\partial y$ and $\partial/\partial z \gg \partial/\partial x$. Such flows are caused by Reynolds stress gradients along the y and z axes, which induce v and w components in the interaction region of the boundary layers. The extent of this wall region in the transverse direction is on the order of (2-4) times the boundary-layer thickness at the given section [6]. Conditions for the appearance and development of such flows with the transition of laminar to turbulent boundary layers in the case of symmetrically developing boundary layers in a straight two-sided corner are studied in detail in [7]. However, in a number of practical cases there is an unsymmetric interaction of boundary layers. Such flows are realized, e.g., in the wing–fuselage junctions and other flight vehicle components. They are characterized by different growth history of the boundary layers on the sides of the corners which leads to asymmetric flow in the neighborhood of the bisecting plane. This type of boundary-layer interaction is the most complex and least studied, and as yet there is no acceptable model for computing such flows. Even experimental data describing the physical picture of the phenomenon are also very limited. The only known investigations are in [8] in which the flow characteristics were studied in the neighborhood of the junction of a slender wing profile mounted on the wall of a wind tunnel test-section of a rectangular cross section. Here the wing leading edge was a semiellipse with $b/a = 1:6$. This paper presents results of experimental studies on the structure of turbulent flow at the junction of two plane surfaces which can be schematically considered as an idealized joint of the wing–fuselage type. A sufficiently wide variation was made in asymmetry which can be conditionally characterized by the ratio of the thicknesses of interacting boundary layers δ_B/δ_A. This made it possible to analyze the variation in the structure of such flows as it transforms from a simple pattern in which symmetrically developing boundary layers ($\delta_B/\delta_A = 1$) interact to form a more complex structure where there is an interaction of boundary layers with different growth histories ($\delta_B/\delta_A > 1$).

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